Vector Mesons and an Interpretation of Seiberg Duality

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We interpret the dynamics of Supersymmetric QCD (SQCD) in terms of ideas familiar from the hadronic world. Some mysterious properties of the supersymmetric theory, such as the emergent magnetic gauge symmetry, are shown to have analogs in QCD. On the other hand, several phenomenological concepts, such as "hidden local symmetry" and "vector meson dominance," are shown to be rigorously realized in SQCD. These considerations suggest a relation between the flavor symmetry group and the emergent gauge fields in theories with a weakly coupled dual description.

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1. Introduction and Summary

The physics of hadrons has been a topic of intense study for decades. Various theoretical insights have been instrumental in explaining some of the conundrums of the hadronic world. Perhaps the most prominent tool is the chiral limit of QCD. If the masses of the up, down, and strange quarks are set to zero, the underlying theory has an $SU(3)_L \times SU(3)_R$ global symmetry which is spontaneously broken to $SU(3)_{diag}$ in the QCD vacuum. Since in the real world the masses of these quarks are small compared to the strong coupling scale,[1] the $SU(3)_L \times SU(3)_R \to SU(3)_{diag}$ symmetry breaking pattern dictates the existence of 8 light pseudo-scalars in the adjoint of $SU(3)_{diag}$. These are identified with the familiar pions, kaons, and eta.[2] The spontaneously broken symmetries are realized nonlinearly, fixing the interactions of these pseudo-scalars uniquely at the two derivative level. See [2], along with references therein, for a systematic exposition of these ideas.

The next hadrons one encounters are the vector mesons, consisting of the rho mesons (with masses around 770 MeV) and their $SU(3)_{diag}$ partners. The analysis of the chiral limit does not place stringent constraints on their dynamics. However, there are strong phenomenological hints of an underlying structure. First of all, to a good approximation, the rho mesons couple equally strongly to pions and nucleons.[3] Secondly, many processes are saturated by vector meson exchanges. This is usually referred to as "vector meson dominance" [3]. Finally, basic parameters associated to the vector mesons (approximately) satisfy curious empirical relations. Perhaps the most striking one [4,5] is $m_\rho^2 = 2g_{\rho\pi\pi}^2 f_\pi^2$, where $g_{\rho\pi\pi}$ is the coupling of the rho meson to two pions.

One can attempt to account for these properties by imagining that the rho mesons (and their $SU(3)_{diag}$ friends) are the gauge fields of a hidden local $[SU(3)]$ gauge symmetry [6,7]. (Of course, the hidden local symmetry $[SU(3)]$ must be higgsed to reproduce the physical nonzero masses of the rho mesons.) Coupling universality may be readily explained by the universality of gauge interactions. The relation $m_\rho^2 = 2g_{\rho\pi\pi}^2 f_\pi^2$ can be interpreted in terms of the usual formula $m_V^2 \sim g^2 v^2$, suggesting that the hidden $[SU(3)]$ symmetry is higgsed at the scale f_π. Lastly, with a little more work, vector dominance can be reproduced

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[1] The approximation of vanishing strange quark mass may seem dubious, but it works pretty well in several circumstances.

[2] Several ideas which can be made precise at large N_c allow to include the eta' in this picture as well, as the Goldstone boson of the axial symmetry [1].

[3] However, the coupling of the rho mesons to themselves is still unknown.

too. This is without doubt a successful phenomenological description of vector mesons. (Another interesting point of view on the subject is given in [8].)

It is appropriate to question the uniqueness (and validity) of the picture outlined above. We know that higgsed gauge symmetries are not physical. Any other way to describe massive spin one particles must yield the same results, for example, one can decide to describe the spin one particles via antisymmetric tensors [9-12]. To construct a map between the various descriptions one must include high dimension operators systematically. Indeed, in the presence of unsuppressed high dimension operators there are absolutely no unique predictions stemming from the existence of a hidden local symmetry. The surprise that QCD offers is that the *minimal* two derivative Lagrangian based on a hidden local symmetry is capable of reproducing a host of phenomena with acceptable precision. We find this rather astounding given that we are not aware of any small parameter that might suppress the high dimension operators.[4]

Such arguments tempt one to conclude that the hidden local symmetry is, in some sense, "real." To prove this one would first need to show that QCD is continuously connected to a theory in which the rho mesons are massless (or just very light for some reason) and that the parameter connecting these theories somehow prevents large corrections from high dimension operators. In this paper we do not attempt to shed any light on this possibility, but we will address in detail a closely related, preliminary, question: *Are there theories in which analogs of the rho mesons are light?* We will argue that supersymmetric QCD in some region of its parameter space is such a theory.

The salient features of the dynamics of SQCD have been understood in a series of outstanding insights [17-23], reviewed in [24]. A lightning review of these results goes as follows. Consider an $[SU(N_c)]$ gauge theory (assuming $N_c > 2$) with N_f flavors. For $N_f = 0$ there are N_c vacua, each of which has a gap of order of the strong coupling scale. For $0 < N_f < N_c$ the theory has no supersymmetric vacua at finite distance in field space and runs away to infinity. For $N_f = N_c$ and $N_f = N_c + 1$ there are moduli spaces of vacua, and the weakly coupled low energy excitations are identified with the gauge invariant operators of the original theory (in other words, the original baryons and mesons). The next phase one encounters is $N_c + 1 < N_f < \frac{3}{2}N_c$. Again, there is a moduli space of vacua. In particular, there is a supersymmetric vacuum at the origin, where all the expectation

[4] Another striking success of a phenomenological approach to QCD is Weinberg's original application of his sum rules [13] (see also [14,15] and the sum rules developed in [16].)

values vanish. However, the infra-red fluctuations cannot be just the baryons and mesons of the original theory; anomaly matching forbids that. Seiberg [23] has managed to identify the low energy fluctuations. These are weakly coupled fields with canonical kinetic terms, consisting of an $[SU(N_f - N_c)]$ IR-free gauge theory with N_f magnetic quarks, and, in addition, a gauge-singlet matrix in the bi-fundamental representation of the flavor group. Obviously, these degrees of freedom are very different from the original variables. The origin of these IR degrees of freedom is shrouded in mystery.

We will see that these magnetic gauge fields associated to $[SU(N_f - N_c)]$ are identified naturally with the familiar rho mesons. Furthermore, the magnetic quarks enter into the story naturally as well. Amusingly, it turns out that SQCD also satisfies vector meson dominance and several other benchmark properties of vector mesons in QCD. Since the rho mesons of SQCD are light (actually massless at the origin of the moduli space) the idea of a hidden local symmetry is on theoretically firm footing, in contrast to the case of QCD. We therefore illuminate some of the mysterious features regarding the dynamics of SQCD in terms of ideas familiar from nuclear physics. SQCD provides an example in which these rough ideas are in fact precise.[5]

The outline of the paper is as follows. In section 2 we briefly review the ordinary theory of pions and introduce vector mesons. Our discussion of these topics is essentially a summary of known results, with incidental original observations. In section 3 we discuss supersymmetric QCD and provide evidence for our main claim. An appendix contains some comments on vector mesons in the large N_c limit of QCD.

2. Pions and Vector Mesons

2.1. Basics of the Theory of Pions

In this subsection we recall how to write Lagrangians for theories with nonlinearly realized symmetries, and we prepare the grounds for the inclusion of vector mesons. For simplicity, we will only discuss the chiral Lagrangian for the breaking of $SU(2)_L \times SU(2)_R \hookrightarrow SU(2)_{diag}$.

[5] Other possible aspects of the similarity between the hidden local symmetry paradigm and Seiberg duality were suggested in [25,26].

The spectrum consists of three pions which are conveniently assembled into a special unitary matrix[6]

$$U = e^{i\pi^a T^a} \ . \tag{2.1}$$

The $SU(2)_L \times SU(2)_R$ symmetry is realized by acting on the matrix U simply as $U' = g_L U g_R^\dagger$. There is a unique invariant Lagrangian at the two derivative level

$$\mathcal{L} = \frac{1}{4} f_\pi^2 Tr\left(\partial_\mu U \partial^\mu U^\dagger\right) \ . \tag{2.2}$$

Note that the diagonal symmetry with $g_R = g_L$ acts linearly on the pions while the axial transformations do not. We can expand the Lagrangian in the number of pions. The first two terms take the form

$$\mathcal{L} = \frac{1}{2} f_\pi^2 \left((\partial\vec{\pi})^2 - \frac{1}{2}\vec{\pi}^2(\partial\vec{\pi})^2 + \cdots \right) \ . \tag{2.3}$$

There is another, equivalent, description of this system that will be more useful for us. The idea is that we can factorize the matrix $U(x)$ in terms of two special unitary matrices ξ_L and ξ_R as follows

$$U(x) = \xi_L(x)\xi_R^\dagger(x) \ . \tag{2.4}$$

This factorization is redundant. The theory has gauge invariance which allows us to redefine $\xi_L \to \xi_L h(x)$, $\xi_R \to \xi_R h(x)$ with any special unitary matrix $h(x)$. The global $SU(2)_L \times SU(2)_R$ symmetry transformation laws are $\xi_L \to g_L \xi_L$, $\xi_R \to g_R \xi_R$. One can rewrite the theory (2.2) in terms of these redundant degrees of freedom as follows

$$\mathcal{L} = -\frac{f_\pi^2}{4} Tr\left[\left(\xi_L^\dagger \partial_\mu \xi_L - \xi_R^\dagger \partial_\mu \xi_R\right)^2\right] \ . \tag{2.5}$$

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[6] Our conventions are

$$T^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} , \qquad T^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} , \qquad T^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} .$$

The following two identities are often useful (we define $\epsilon^{123} = 1$)

$$T^a T^b = \delta^{ab} + i\epsilon^{abc} T^c \ ,$$

$$e^{i\pi^a T^a} = \cos\left(\sqrt{\vec{\pi}^2}\right) + i\frac{\pi^a T^a}{\sqrt{\vec{\pi}^2}} \sin\left(\sqrt{\vec{\pi}^2}\right) \ .$$

It is easy to check that this Lagrangian is gauge invariant and it is also invariant under global symmetry transformations [27].

The physical properties of the pions can be calculated by fixing a gauge. For example, we can choose to fix a gauge in which $\xi_L = \xi_R^\dagger$. Global symmetry transformations take us out of this gauge, but we can always reinstate our gauge choice by an accompanying gauge transformation.

It is useful to think of this theory in the following language. The $[SU(2)]$ gauge symmetry endows the model with a quiver-like structure $SU(2)_L \times [SU(2)] \times SU(2)_R$, where ξ_L is in the bi-fundamental of $SU(2)_L \times [SU(2)]$ and ξ_R is in the bi-fundamental of $[SU(2)] \times SU(2)_R$. The vacua of this theory are parametrized by constant matrices ξ_L, ξ_R, modulo gauge transformations. So we can always choose $\xi_L = 1$ and ξ_R is a general special unitary matrix. This VEV for ξ_L breaks the gauge symmetry but a diagonal flavor symmetry coming from a mixture of the global transformations in $SU(2)_L$ and (global) gauge transformations in $[SU(2)]$ remains. (This pattern of flavor generators mixing with gauge generators will be a recurring theme.) Then, the VEV for ξ_R breaks the flavor symmetry to $SU(2)_{diag}$. Note that so far in this model there is a gauge symmetry but no gauge fields.

2.2. Adding Gauge Fields

The second version of the theory of pions (2.5) has a redundancy but no gauge fields associated to this redundancy. Consider adding such a triplet of real vector fields ρ_μ^a transforming as usual

$$\rho_\mu \equiv \rho_\mu^a T^a \to h^\dagger \rho_\mu^a T^a h + i h^\dagger \partial_\mu h \ . \tag{2.6}$$

We can construct two natural objects transforming homogeneously under (2.6)

$$\rho_\mu^L = \rho_\mu - i\xi_L^\dagger \partial_\mu \xi_L \ , \qquad \rho_\mu^R = \rho_\mu - i\xi_R^\dagger \partial_\mu \xi_R \ . \tag{2.7}$$

At the two derivative level the most general Lagrangian symmetric under $L \leftrightarrow R$ can be written as

$$\mathcal{L} = -\frac{1}{g^2}(F_{\mu\nu}^a)^2 + \frac{f_\pi^2}{4} Tr\left[\left(\rho_\mu^L - \rho_\mu^R\right)^2\right] + a\frac{f_\pi^2}{4} Tr\left[\left(\rho_\mu^L + \rho_\mu^R\right)^2\right] \ . \tag{2.8}$$

So far, a, g are undetermined real parameters. We will see that this theory includes a massive spin one particle with mass of order $g f_\pi$, so we should discuss its regime of validity.

First of all, an effective action for massive particles is subtle since, using the equations of motion, operators with different numbers of derivatives can mix. Secondly, for massive particles, one needs a small parameter that justifies truncating the effective action to include finitely many terms. Therefore, for now, to make sense of the effective theory (2.8) we will assume that the gauge coupling g is parametrically small. (Conversely, when the spin 1 fields are parametrically light, one must use gauge theories for consistency.) Eventually, we would like the massive spin one particles in the theory above to be identified with the rho mesons of QCD.[7] In nature, the gauge coupling of the rho mesons is by no means small. In spite of this, the theory (2.8) reproduces some of the properties of QCD remarkably well.

Note that $a = 1$ is special in (2.8). In this case ξ_L interacts with ξ_R only through gauge fields. As a consequence, when $g = 0$, the global symmetry is enhanced due to the global gauge transformations to $SU(2)_L \times SU(2)^2 \times SU(2)_R$. This symmetry argument has led Georgi [28,29] to propose the importance of $a = 1$. It is an inspiring idea, but unfortunately, the resulting theory does not seem to describe QCD. We will see that QCD is best described by a different, also special, value of a.

Denoting $\xi_L = e^{i\pi_L^a T^a}$, $\xi_R = e^{i\pi_R^a T^a}$ and expanding (2.8) to quadratic order we get

$$\mathcal{L} = -\frac{1}{g^2}(F_{\mu\nu}^a)^2 + \frac{f_\pi^2}{2}\left(\partial_\mu(\pi_L^a - \pi_R^a)\right)^2 + \frac{af_\pi^2}{2}\left(\partial_\mu(\pi_L^a + \pi_R^a) + 2\rho_\mu^a\right)^2 + \cdots \ . \tag{2.9}$$

In order to find the physical spectrum we pick unitary gauge $\pi_L = -\pi_R \equiv \pi$. We find a triplet of massless pions and a massive gauge field with mass

$$m_\rho^2 = ag^2 f_\pi^2 \ . \tag{2.10}$$

More generally, the interesting terms in the interacting Lagrangian in this unitary gauge can be easily calculated (and simplified by using the free equations of motion and integration by parts)

$$\mathcal{L} = -\frac{1}{g^2}(F_{\mu\nu}^a)^2 + 2f_\pi^2\left((\partial_\mu\pi)^2 - 2\pi^2(\partial_\mu\pi)^2\right) + 2af_\pi^2\left(\epsilon^{abc}\pi^a\partial_\mu\pi^b + \rho_\mu^c\right)^2 + \cdots \ . \tag{2.11}$$

The equation of motion of the massive gauge field sets it at low energies to $-\epsilon^{abc}\pi^a\partial_\mu\pi^b$. Plugging this into (2.11) and comparing with (2.3) we verify that f_π is indeed correctly identified as the pion decay constant.

[7] We are ignoring the axial vector mesons for simplicity.

We would like to evaluate the global $SU(2)_{diag}$ currents $(\mathcal{J}^a_{diag})_\mu$. This calculation is best done before fixing a gauge, in order to guarantee that our expression in unitary gauge descends from a gauge invariant operator. The complete gauge invariant expression for the conserved current is

$$(\mathcal{J}^a_{diag})_\mu = \frac{f_\pi^2}{4} Tr\left[\left(\rho^L_\mu - \rho^R_\mu\right)\left(\xi_L^\dagger T^a \xi_L - \xi_R^\dagger T^a \xi_R\right)\right] + \frac{a f_\pi^2}{4} Tr\left[\left(\rho^L_\mu + \rho^R_\mu\right)\left(\xi_L^\dagger T^a \xi_L + \xi_R^\dagger T^a \xi_R\right)\right] . \tag{2.12}$$

In unitary gauge the expression above becomes

$$(\mathcal{J}^a_{diag})_\mu = 2a f_\pi^2 \rho^a_\mu + 2f_\pi^2(a-2)\epsilon^{abc}\pi^b\partial_\mu\pi^c + \text{three particles} + \cdots . \tag{2.13}$$

The immediate lessons from this formula are twofold. First, we see that upon setting $a = 2$, the coefficient of the second term vanishes. The fact that $a = 2$ is special is inconspicuous in the original Lagrangian (2.8). However, we will see that $a = 2$ is actually the value which best describes the phenomenology of QCD. A second corollary is that there is a general relation between the physical coupling of the rho meson to pions $g_{\rho\pi\pi} \equiv \frac{1}{2} g a$, and the amplitude with which the current creates a photon $g_{\rho\gamma} \equiv g a f_\pi^2$,

$$g_{\rho\gamma} = 2 g_{\rho\pi\pi} f_\pi^2 . \tag{2.14}$$

Note that the unknown parameters a, g cancel from this relation. Indeed, this relation was recognized very early on via current algebra techniques by KSFR [4,5] (also reviewed, for example, in [11]). To derive this relation we assumed that g is very small, but one can brazenly test this relation in QCD. The agreement is about 10%, which is remarkable.

The next two lessons to draw from (2.11),(2.13) are a little less straightforward. One should study the electromagnetic form factor of the charged pion. We prepare a charged pion with four-momentum p at early times which is then struck by an off-shell photon. The pion eventually leaves the interaction point with four-momentum p'. To calculate the result of this process we evaluate the electromagnetic current matrix element (denoting $q = p' - p$)

$$\langle \pi(p) \left| J_\mu^{QED}(0) \right| \pi(p') \rangle = (p + p')_\mu F(q^2) . \tag{2.15}$$

Equation (2.15) follows from the masslessness of the pion and current conservation. From (2.13) we see that at tree-level the process has two components: a direct contact term between the current and the pions or an emission of a rho meson which propagates as a virtual particle and then, using the vertex $\sim \epsilon^{abc}\rho^{\mu a}\pi^b\partial_\mu\pi^c$, hits the target. These

two contributions have a different analytic form. One more thing we know is that as a consequence of the fact that the pion has charge 1, in the deep infrared the form factor is independent of a and satisfies $F(q^2 = 0) = 1$.

We find

$$F(q^2) = (1 - \frac{1}{2}a) + \frac{\frac{1}{2}am_\rho^2}{m_\rho^2 - q^2} \ . \tag{2.16}$$

We can now start to appreciate why the special point $a = 2$ is called the point of "vector dominance." It is because the effect of scattering a photon on a pion target is fully accounted for by a ρ exchange. Said in other words, the photon and rho gauge boson are maximally mixed. The fact that this process is saturated by ρ meson exchange also implies that a series of other processes is controlled by ρ mesons,[8] but we will not discuss this here. Note that choosing $a = 2$, the general relation (2.10) becomes in terms of $g_{\rho\pi\pi}$

$$m_\rho^2 = 2g_{\rho\pi\pi}^2 f_\pi^2 \ . \tag{2.17}$$

This is another relation that is obeyed by the real world (at a level of around 5%).

One can provide a rationale for choosing $a = 2$ by an argument akin to the Weinberg sum rules. Asymptotic freedom tells us that in the deep Euclidean region the form factor drops like a power

$$\lim_{q^2 \to -\infty} F(q^2) \sim \frac{1}{q^2} \ . \tag{2.18}$$

Therefore by extending the form factor to an analytic function in the complex q^2 plane and integrating $F(q^2)/q^2$ over a large contour we get zero. This means that the integral of the imaginary part over the time-like domain is zero. Following Weinberg's original derivation of his sum rules [13], we ignore all the contributions besides those associated to the light resonances. Then, considering some contour γ that encircles the origin and the rho meson pole we get

$$\int_\gamma d(q^2) \frac{F(q^2)}{q^2} = 0 \ . \tag{2.19}$$

Since $F(0) = 1$ and the residue at $q^2 = m_\rho^2$ follows from (2.16), we arrive at

$$a = 2 \ . \tag{2.20}$$

[8] Another miracle that happens for $a = 2$ is that the coupling of three rho mesons is of the same strength as the coupling to two pions. This is the famous coupling universality hypothesis in QCD.

One has thus established vector meson dominance. (Vector dominance is often regarded as an input. Our point of view is that it is not a random fact about the hadronic world, rather, under some circumstances it could have been predicted by sum rules, in the same way that Weinberg predicted the axial vector mesons.)

We can even *estimate* the numerical value of the dimensionless parameter $g_{\rho\pi\pi}$. For this we have to consider the function $F(q^2)$ itself, as function of a complex variable. The coefficient of $1/q^2$ in (2.18) can be calculated by explicit Feynman diagrams. In fact, the coefficient logarithmically runs to zero at infinitely large energies. Since this decay of the coefficient is only logarithmic, to correctly utilize the fact that

$$\int_{\infty} d(q^2)F(q^2) = 0 \ , \tag{2.21}$$

it is most convenient to read out the coefficient of $1/q^2$ in a region where asymptotic freedom already dominates but the logarithmic running has not been substantial. The experimental results [30,31] suggest that Bjorken scaling is approximately true already at a few GeV and the coefficient of $1/q^2$ is around 0.45 GeV^2. This is again related to a sum over resonances by the Cauchy theorem. The low energy contribution to this integral comes from the residue of the pole at the ρ meson mass. The contour argument relates these two residues given that we neglect the heavy mesons (and multi-particle states). This, together with the previous result $a = 2$, allows us to conclude that $g_{\rho\pi\pi} \sim 5.1$. The correct value is around 6. This estimate is as impressively successful as many of the results obtained via sum rules.

2.3. Summary

Let us summarize some of the important points we discussed in this section. It will be important to keep these in mind for our discussion of supersymmetric QCD.

1. Vector mesons are included in the chiral Lagrangian by splitting the pion field into two redundant pieces and adding gauge fields for this redundancy. Then a sequence of symmetry breaking phenomena takes place. First, the gauge symmetry is broken and the full $SU(2)_L \times SU(2)_R$ flavor symmetry survives as a linear combination of flavor generators and gauge generators. Subsequently, the flavor symmetry is further broken to $SU(2)_{diag}$.
2. This description makes physical sense only for small values of the gauge coupling (with fixed f_π), but it seems to describe many important properties of nature in spite of the fact that the gauge coupling of the ρ mesons in nature is pretty large.

3. The ρ mesons can be created from the vacuum by the action of unbroken flavor symmetry generators.
4. In the framework of the two derivative effective field theory, the relation (2.14) follows. If one further applies a sum rule in the usual way, one also finds vector meson dominance (meaning that the ρ mesons can fully account for various physical effects such as the form factor we investigated) and the second KSFR relation (2.17). Note that if, in some sense, effects from heavier states in QCD are small, then treating the theory as if higher derivative terms are less important than the leading ones, and applying the sum rule, would both be justified. The experimental success of the results obtained suggests that the effects of heavier states are indeed small.
5. There is no Higgs field that accompanies the massive ρ mesons. However, in theories with light rho mesons, we surely expect to find Higgs fields. We will see in the next section that this is indeed what happens in supersymmetric QCD.

3. Supersymmetric Quantum Chromodynamics

We consider $SU(N_c)$ gauge theory with N_f flavors $Q^i, \widetilde{Q}_i$, $i = 1...N_f$. Our interest lies mostly in the IR-free non-abelian phase of SQCD, $N_c+1 < N_f < \frac{3}{2}N_c$. The symmetry group is $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$, where the non-anomalous R-symmetry is $U(1)_R(Q) = U(1)_R(\widetilde{Q}) = 1 - N_c/N_f$.

At energies much below the strong coupling scale, this theory flows to the Seiberg dual [23] $SU(N_f - N_c)$ IR-free gauge theory with N_f magnetic quarks $q_i, \widetilde{q}^i$ and a gauge-singlet matrix M^i_j in the bi-fundamental representation of the flavor group. Obviously, these degrees of freedom are very different from the original variables, but the vacua agree upon introducing the superpotential

$$W = \widetilde{q}^j M^i_j q_i \ . \tag{3.1}$$

In addition, the deformations of the two theories agree. (And the anomalies of course match.)

For small VEVs, these "magnetic" fields have a canonical Kähler potential, albeit with an unknown normalization. Neither the $SU(N_f - N_c)$ magnetic gauge fields nor the magnetic quarks appear as well defined local operators in the UV. This must be so because they are charged under a hidden local symmetry group, so they are not gauge invariant.

Our goal here is to establish a dictionary (or an analogy) between ideas familiar in ordinary QCD and the low energy description of supersymmetric QCD. We will provide evidence for the claim that the gauge fields should be thought of as rho mesons and the magnetic quarks are analogous to ξ_L, ξ_R. As we have already mentioned, we will see that many ideas that in QCD work well phenomenologically but are hard to justify theoretically can, in fact, be justified in the supersymmetric version. Another benefit of taking this analogy seriously is that many hitherto mysterious features of SQCD can be understood as cousins of familiar ideas from QCD.

3.1. On the Moduli Space

At the origin of the moduli space of SQCD the magnetic gauge fields are massless, so to test our proposal we need to move away (slightly) from the origin. Let us consider the following direction in moduli space

$$q \equiv \begin{pmatrix} \chi_{(N_f-N_c)\times(N_f-N_c)} \\ \varphi_{N_c\times(N_f-N_c)} \end{pmatrix} = v \begin{pmatrix} 1 & 0 & ... & 0 \\ 0 & 1 & ... & 0 \\ . & . & . & . \\ . & . & . & . \\ 0 & 0 & ... & 1 \\ 0 & 0 & ... & 0 \\ . & . & . & . \\ . & . & . & . \end{pmatrix} . \tag{3.2}$$

To study the physics along this flat direction it is also convenient to decompose the other magnetic quark and meson

$$\widetilde{q} \equiv \left(\widetilde{\chi}_{(N_f-N_c)\times(N_f-N_c)}, \quad \widetilde{\varphi}_{(N_f-N_c)\times N_c} \right) , \qquad M \equiv \begin{pmatrix} X_{(N_f-N_c)\times(N_f-N_c)} & Y \\ \widetilde{Y} & Z_{N_c\times N_c} \end{pmatrix} . \tag{3.3}$$

Along this flat direction, the symmetry is broken from the original global symmetry $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$ to $SU(N_f - N_c)_L \times SU(N_c)_L \times SU(N_f)_R \times U(1)'_B \times U(1)'_R$.[9] Therefore, the breaking along this baryonic branch is essentially of the form

$$SU(N_f)_L \hookrightarrow SU(N_f - N_c)_L \times SU(N_c)_L \ . \tag{3.4}$$

The massless particles consist of $2N_f N_c - 2N_c^2 + 1$ Goldstone bosons (one of them is just the expectation value v itself) and the massless mesons Y, Z. The IR theory along this

[9] The primes on the various $U(1)$ groups mean that they survive by mixing with non-Abelian flavor generators and perhaps among themselves.

moduli space does not include massless gauge fields and it is therefore a simple IR-free nonlinear sigma model for the coset (3.4) coupled to the massless mesons.[10]

For small v the magnetic dual variables allow us to describe the correct light (but not necessarily massless) excitations around this flat direction. The dual magnetic gauge theory $[SU(N_f - N_c)]$ is completely higgsed, hence, for small v, there are light massive gauge fields along this flat direction with mass scaling like $\sim gv$. In addition, the mesons $X, \widetilde{Y}$ as well as the magnetic quarks $\widetilde{q}$ are all massive (but light) with mass of order v.

Already at this stage we see some *superficial* hints for the correspondence we are proposing. First of all, the flavor symmetry $SU(N_f - N_c)_L$ survives at very low energies because in the magnetic description it mixes with global gauge transformations.[11] In other words, the Goldstone bosons for the breaking (3.4) are the magnetic quarks φ_i^c. The index c transforms in the fundamental representation of the unbroken $SU(N_f - N_c)_L$ flavor symmetry, but it really descends from a gauge index at higher energies. Yet another way to say the same thing is that given the nonlinear sigma model for $SU(N_f)_L / \left(SU(N_f - N_c)_L \times SU(N_c)_L\right)$ the way we could reintroduce the magnetic gauge fields into this theory is by promoting the redundant $SU(N_f - N_c)_L$ transformations into a local symmetry and then adding spin one particles with kinetic terms. This is precisely the way we introduced the rho mesons into the pion Lagrangian in section 2. We see that the magnetic quarks are also natural players in the story, they are the redundant variables we add to allow for a local symmetry, hence, they are analogous to the ξ_L, ξ_R degrees of freedom in the theory of the previous section. (One difference is that SQCD also contains the Higgs fields associated to global symmetry breaking. As mentioned in subsection 2.3, this must have been the case since in SQCD a limit in which the global symmetry is restored exists.)

So far these are merely intuitive similarities, but they can be made precise by studying the global symmetry currents of the theory (3.1). Consider the $SU(N_f - N_c)_L$ global symmetry current superfields. We can attempt to write them in terms of the magnetic dual variables. However, since we do not know the normalizations of the (canonical)

[10] This sigma model also matches the one obtained from the electric theory, as was described in [32].

[11] This mixing, which occurs naturally in SQCD, has been recently used for phenomenological purposes [33]. Some related discussions can be found in [34-36]. It would be nice to see if there are connections between our approach and the analysis in [37,38], where this color-flavor locking phenomenon played a major role.

kinetic terms, there are some order one numbers that we cannot control (we do know their signs though). This will not affect our discussion, and we will henceforth simply suppress these incalculable (positive) numbers. The expression for the currents then takes the form ($V \equiv V^a T^a$ is the magnetic vector superfield)

$$\mathcal{J}^a_{SU(N_f-N_c)_L} = \chi^c_j (e^V)^d_c (\chi^\dagger)^i_d (T^a)^j_i - X^i_l (X^\dagger)^l_j (T^a)^j_i - (\widetilde{Y})^i_l (\widetilde{Y}^\dagger)^l_j (T^a)^j_i \ . \tag{3.5}$$

The meaning of this expression is most transparent once we fix a unitary gauge for the magnetic group. Unitary gauge means that we set all the quadratic terms mixing the gauge field and the matter fluctuations to zero. This is achieved by

$$\forall a. \ \ \langle \chi^c_i \rangle (T^a)^d_c (\delta \chi^\dagger)^i_d = 0 \ . \tag{3.6}$$

(The general theory of such unitary gauges has been developed in [39].) In our case, (3.2), $\langle \chi^c_i \rangle = v \delta^c_i$ and unitary gauge (3.6) therefore just means that $\delta \chi^c_i \sim \delta^c_i$. In other words, the only physical degree of freedom in the χ magnetic quarks is the overall scale v, which is one of the complex Goldstone bosons. We denote this special mode by π, i.e. $\chi^c_i = v\delta^c_i + \pi \delta^c_i$.

Evaluating the current (3.5) in this gauge and dropping all terms with more than two particles we find

$$\mathcal{J}^a_{SU(N_f-N_c)_L} = v^2 (e^V)^d_c (T^a)^c_d + v(\pi + \pi^\dagger)(e^V)^d_c (T^a)^c_d - \text{mesons} + \cdots \ . \tag{3.7}$$

The meson terms that we suppressed are identical to those in (3.5). The physical quantity we would like to compute is the form factor of the Goldstone bosons φ. We will be interested only in the tree-level contributions. The terms quadratic in mesons surely cannot contribute, since there is no way to draw a diagram at tree level. Similarly, the second term in (3.7) plays no role. We remain with the first term, in which only the piece linear in V contributes at tree level. This gives

$$F_\varphi(q^2) = \frac{m_V^2}{m_V^2 - q^2} \ . \tag{3.8}$$

Since a term quadratic in φ is absent from (3.7), there is no constant piece in the form factor.

We see that not only is the identification between the Seiberg dual gauge fields and the rho mesons manifest, SQCD also satisfies vector dominance. So SQCD seems to sit at a point analogous to $a = 2$ in QCD (which, as we explained, is closest to describing

nature). That a is equal 2 in QCD has been motivated by sum rules. However, in SQCD, the analogous result follows rigorously from Seiberg duality.

Note that at energies below the massive vector boson the form factor is 1, as it should be. At such low energies we should integrate out the massive vector field, and the massive fields $\widetilde{q}$, X, $\widetilde{Y}$. Note that as far as X, $\widetilde{Y}$ are concerned, they do not have quadratic mixing terms with light fields and so the last two terms in (3.5) cannot produce terms quadratic in the light fields. We can thus ignore these terms. What remains is to solve the equations of motion of the massive vector fields. In our unitary gauge we get an expansion in the number of light fields, starting with the following quadratic term in the Goldstone bosons

$$V^a \sim \varphi_i^c (T^a)_c^d (\varphi^\dagger)_d^i \ . \tag{3.9}$$

Plugging this back into the expression for the current (3.5), we find that at low energies the $SU(N_f - N_c)_L$ current is

$$\mathcal{J}^a_{SU(N_f-N_c)_L} \sim \varphi_i^c (T^a)_c^d (\varphi^\dagger)_d^i + \text{three particles} + \cdots \ . \tag{3.10}$$

This is, of course, the expected result, since φ transforms linearly under $SU(N_f - N_c)_L$ transformations at very low energies. The physical interpretation of what is happening here is as follows. At energies above v the fields φ are not charged under the global $SU(N_f - N_c)_L$ symmetry transformations and this is why they are absent from (3.5). However, due to higgsing of the magnetic gauge symmetry and the fact that $SU(N_f - N_c)_L$ is realized as a mixture of the original flavor transformations and some global gauge transformations, φ is indeed charged under $SU(N_f - N_c)_L$ at low energies. This is why there is a quadratic term in φ in the expression for the $SU(N_f - N_c)_L$ current at low energies (3.10). Since φ has such an "energy dependent charge," vector meson dominance is realized in earnest: at high energies the form factor is saturated by a gauge field propagator, but at very low energies it is accounted for by the structure-less charged particle φ.

We thus see that SQCD slightly deformed from the origin has a rich structure that in some respects resembles QCD, especially in the way vector mesons appear and the way they dominate physical processes. The conclusion that the Seiberg dual gauge fields are analogous to the ρ mesons, and the magnetic quarks' role is similar to those of ξ_L, ξ_R, is unavoidable. In addition to the structural similarities, phenomenological properties such as vector dominance, and relations analogous to (2.14),(2.17), are satisfied.

One general comment is in order. In (the chiral limit of) QCD the full unbroken global symmetry creates vector mesons from the vacuum and they are pretty heavy. In SQCD, on the branch of moduli space we have studied here, the unbroken symmetry contains $SU(N_f - N_c)_L \times SU(N_c)_L$. We have shown that the currents of $SU(N_f - N_c)_L$ create light vector mesons from the vacuum, but $SU(N_c)_L$ has not played a major role. Indeed, we expect it to create heavy one particle states, therefore not visible in the Seiberg dual description; in the IR the $SU(N_c)_L$ currents only create states with more than one particle. Interestingly, the roles of $SU(N_f - N_c)_L$ and $SU(N_c)_L$ get reversed in a different region of parameter space.

Consider taking $N_f > 3N_c$. In this case the electric theory is not UV free but the magnetic theory (3.1) is. The flat direction (3.2) still exists but now for small v we cannot analyze it in terms of the magnetic variables since they are strongly coupled. The electric variables provide the weakly coupled description, and the $[SU(N_c)]$ gauge theory is now the hidden local symmetry which one encounters in the IR. It is not hard to see that now the $SU(N_c)_L$ flavor symmetry survives in the IR because of mixing with global gauge transformations and that the currents of $SU(N_c)_L$ excite the light gauge bosons of the hidden local symmetry.

We therefore see that the theory we are discussing has both light and heavy rho mesons. The light rho mesons are created by some global symmetry currents and the heavy ones are created by a different set of global symmetry currents. We have no theoretical control over the heavy rho mesons, but we can say a lot about the parameterically light ones. A similar picture will emerge when we study SUSY-breaking field configurations in the next subsection.

3.2. Off the Moduli Space

We would like to subject the identification we are proposing to further tests. The idea is that the role of the dual gauge fields as rho mesons *must* manifest even in small deformations of the theory that may have a (small) nonzero vacuum energy density.

We consider again the free magnetic phase $N_c + 1 < N_f < \frac{3}{2} N_c$ and add a mass term to all the electric quarks

$$W_{electric} = mQ^i \widetilde{Q}_i \ . \tag{3.11}$$

The dynamics of this theory for small field VEVs has unfolded only in recent years, starting with [40]. The symmetry group of (3.11) is $SU(N_f)\times U(1)_B$. The description of the theory near the origin is in terms of the Seiberg dual variables

$$W_{magnetic} = \widetilde{q}^j M^i_j q_i - \mu^2 M^i_i \ , \tag{3.12}$$

where we have denoted $\mu^2 = -m\Lambda$. The symmetry group of the theory contains $SU(N_f)$, but we definitely do not see N_f^2-1 gauge bosons in the IR, just $(N_f-N_c)^2-1$ of them. This apparent contradiction is resolved by carefully studying the dynamics of (3.12).

The F-term equations for the meson field $q^j q_i - \mu^2\delta^j_i = 0$ cannot all be satisfied because the rank of the first term is necessarily smaller than N_f. This means that for small field VEVs there are no SUSY vacua and the vacuum energy density is (at least) of order μ^4 (we choose m,Λ to be real without loss of generality). We can trust these non-supersymmetric configurations as long as the typical VEVs and energy densities are much smaller than the cutoff. For this reason we focus on the regime $m \ll \Lambda$ (which also implies $\mu \ll \Lambda$).[12]

Using the same notation as in (3.2) and (3.3), one finds that the classical energy density is minimized by setting $\chi^c_i = \mu\delta^c_i$, $\widetilde{\chi}^i_c = \mu\delta^i_c$, while all the other fields are set to zero. The symmetry is broken as follows

$$SU(N_f)\times U(1)_B \hookrightarrow SU(N_f-N_c)\times SU(N_c)\times U(1)'_B \ . \tag{3.13}$$

We now see how the apparent contradiction mentioned above is going to be resolved. The crux of the matter is that, in the vacuum close to the origin, the massive theory (3.11) spontaneously breaks the $SU(N_f)$ global symmetry. Unlike supersymmetric vacua, in which the pattern of symmetry breaking can be inferred at weak coupling, in this case one must genuinely use the duality. There is no known way to anticipate (3.13) from the electric description. On the other hand, that the symmetry breaks in this way is absolutely necessary for the consistency of the picture we are proposing.[13]

From here the story proceeds in parallel to the story in QCD. The main difference being that some unbroken generators (those of $SU(N_f-N_c)$) create light rho mesons from the vacuum, namely, the Seiberg dual gauge fields. The other symmetry generators only

[12] At field VEVs much larger than $\sim\mu$ there are in fact supersymmetric vacua, but they are of no interest to us here.

[13] We thank M. Strassler for a helpful conversation on the subject.

create massive particles, not visible at low energies. This is not a contradiction, it only means that SQCD in this SUSY-breaking vacuum has two distinct sets of rho mesons. Those associated to $SU(N_f - N_c)$ are very light and can in fact be continuously taken to be massless, while those of $SU(N_c)\times U(1)'_B$ are heavy and cannot be analyzed analytically. Our main point is that the existence of this set of arbitrarily light rho mesons allows, among other things, to exhibit some phenomenological ideas from QCD in a rigorous setup.

The Goldstone bosons bassociated with (3.13) are given by $\varphi + \widetilde{\varphi}^*$ and an additional real singlet mode $Tr(Im(\chi - \widetilde{\chi}))$. One also finds the so called "pseudo-moduli," which parametrize classical non-compact flat directions. In our case (3.12), one-loop effects have been shown [40] to set these fields to zero.

As we have already implied, the currents of the unbroken $SU(N_f - N_c)$ symmetry are candidates for creating the magnetic gauge fields from the vacuum. We can write an explicit expression (up to an overall coefficient) for the $SU(N_f - N_c)$ currents

$$\mathcal{J}^a_{SU(N_f-N_c)} = \chi^c_j (e^V)^d_c (\chi^\dagger)^i_d (T^a)^j_i - (\widetilde{\chi}^\dagger)^c_j (e^{-V})^d_c \widetilde{\chi}^i_d (T^a)^j_i + mesons\ , \tag{3.14}$$

where we have suppressed all the terms bilinear in mesons. The natural choice of unitary gauge in this case is

$$\forall a.\ \ \langle \chi^c_i \rangle (T^a)^d_c (\delta\chi^\dagger)^i_d - \langle \widetilde{\chi}^i_d \rangle (T^a)^d_c (\delta\widetilde{\chi}^\dagger)^c_i = 0\ . \tag{3.15}$$

Plugging the VEVs of χ, $\widetilde{\chi}$, we see that unitary gauge amounts to setting

$$\delta\chi - \delta\widetilde{\chi} \sim \mathbb{I}\ . \tag{3.16}$$

Similarly to the discussion after (3.6), the unit matrix corresponds to some complexified transformations of the vacuum expectation values. Since in this case the vacuum is not supersymmetric, only the imaginary part of (3.16) is in fact massless while the real part is one of the pseudo-moduli that obtains a mass from radiative corrections.

In this gauge the current multiplet becomes

$$\mathcal{J}^a_{SU(N_f-N_c)} = \mu^2 V^a + \text{two particles} + \cdots\ . \tag{3.17}$$

Hence, we see again that the unbroken $SU(N_f - N_c)$ currents create the magnetic gauge fields from the vacuum. Since at energy scales of order μ and above a quadratic term $\sim \varphi T^a \varphi^\dagger$ is absent from (3.17), the form factor for the Goldstone bosons $\varphi + \widetilde{\varphi}^*$ will be of the form (3.8), satisfying vector dominance. At low energies we can integrate out the massive vector field and the current becomes quadratic in the Goldstone bosons. The physics of this is identical to what we have found in the previous subsection. This completes our analysis of the SUSY-breaking case, corroborating our proposal.

3.3. Open Questions

It would be nice to check whether the phenomena we found here are general or not. To address this question, one would need to study other examples, such as the orthogonal and symplectic cases. More exotic examples, such as adjoint SQCD [41-43], are also interesting to study.

In general, it may be possible to identify the emergent gauge bosons with rho vector mesons only if the number of emergent gauge bosons is not larger than the number of flavor symmetry generators. Thus, we obtain an inequality between two objects which seem unrelated at first sight. Clearly, if vector-like theories (in the IR-free phase) that violate this inequality exist, they must be truly exotic.[14] It would be interesting to conduct a systematic survey of the known models, but here we will merely check this inequality in adjoint SQCD. From our point of view, the latter is a curious example because the rank of the emergent group can be arbitrarily larger than the rank of the global symmetry group, so if the inequality is to hold water, there must be a nontrivial interplay between the location of the IR-free window, the rank of the emergent gauge group, and the rank of the flavor group.

Consider SQCD with gauge group $SU(N_c)$, N_f electric quarks, and an adjoint field X. We include the superpotential

$$W = Tr(X^{k+1}) \ . \tag{3.18}$$

The matter fields in the dual theory [42] consist of a magnetic gauge group $SU(kN_f - N_c)$, N_f magnetic quarks, an adjoint field, and a family of gauge singlets. Therefore, the dual theory is IR-free as long as the beta function

$$-\beta_{dual} = (2k-1)N_f - 2N_c \ , \tag{3.19}$$

is positive. In addition, we must take $N_f \geq N_c/k$ (for $N_f < N_c/k$ the theory has no vacuum). We conclude that the IR-free phase is realized for

$$\frac{N_c}{k} \leq N_f < \frac{2N_c}{2k-1} \ . \tag{3.20}$$

---

[14] It is worth mentioning that there may be supersymmetric theories in which the analogs of the axial vector mesons and perhaps also the ρ' mesons are massless. In the case of ρ', by definition, one global symmetry current can create both the ρ and ρ' from the vacuum. Naively, it seems that in this case the magnetic gauge group would have to be a product group and so a more refined version of our inequality would still hold. Needless to say, it would be interesting to search for such theories and to investigate their properties.

To test our inequality, it is sufficient to show that throughout this window $kN_f - N_c < N_f$. Indeed, this is equivalent to satisfying $N_f < N_c/(k-1)$, which is consistent with (3.20).

This is just a preliminary necessary condition for our proposal to be realized in adjoint SQCD. It would be nice to work out the details in these and other theories.

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Appendix A. The Large N_c Limit

We would like to offer some comments on vector mesons in QCD with many colors $N_c \to \infty$ [44]. We start by recalling some well known facts about this limit (see [45] for more details). At large N_c, the two point function of, say, flavor currents can be decomposed into a sum over all the meson states, which are exactly stable (and free) at infinite N_c. Since this is of order N_c, the amplitude to create a meson from the vacuum by the action of a current is $\sqrt{N_c}$. From this it follows that the pion decay constant scales like $f_\pi \sim \sqrt{N_c}$. The masses of these states do not scale with N_c. Similarly, by considering properties of three point functions, we learn that cubic interaction vertices are suppressed by $1/\sqrt{N_c}$. Importantly for us, this also shows that currents can create two particle states from the vacuum with amplitude of order 1.

Let us test this picture against the effective theory of ρ mesons (2.11),(2.13). From the interaction of three ρ mesons we immediately conclude that $g \sim \frac{1}{\sqrt{N_c}}$. We then observe from the interaction of two pions and ρ that the parameter a must stay finite in the large N_c limit. Note that this implies through (2.10) that the mass of the ρ meson stays finite for large N_c. From (2.13) we see that the amplitude to create a single rho meson from the vacuum scales like $gf_\pi^2 \sim \sqrt{N_c}$, which is consistent with the general large N_c rule. From (2.13) we also see that a pair of pions is created with amplitude of order 1, in accordance with large N_c expectations. We therefore conclude that the theory of rho

mesons is consistent with our expectations from large N_c, but large N_c arguments do not fix a.

Let us now study the pion form factor, including the whole tower of vector mesons indexed by n. One can then imagine that the unbroken flavor current contains all these mesons and possibly a bilinear term $\pi\partial\pi$ like in (2.13). The form factor of the pion then takes the form

$$F(q^2) = c + \sum_n \frac{\kappa_n}{q^2 - m_n^2} \ . \tag{A.1}$$

The constant c comes from the possibility of the current to create a two pion state from the vacuum directly. The κ_n are some dimension two coefficients which depend on the amplitude of the current to create the nth vector meson from the vacuum and also on the vertex which connects the nth vector meson with two pions. Thus, both c, κ_n are finite for $N_c \to \infty$. We need to make sure that $F(0) = 1$, which gives one relation among the infinitely many coefficients in (2.16). Assuming asymptotic freedom, one can write a sum rule $\int_\infty F(q^2)/q^2 = 0$ which then implies (to arrive at the formula below we use the fact that the residue at the origin is 1)

$$1 + \sum_n \frac{\kappa_n}{m_n^2} = 0 \ . \tag{A.2}$$

Together with the constraint $F(0) = 1$ we find that $c = 0$. Hence, under our assumptions, the form factor at large N_c in theories which are asymptotically free must be of the resonance-saturated form

$$F(q^2) = \sum_n \frac{\kappa_n}{q^2 - m_n^2} \ . \tag{A.3}$$

This form is manifest in many examples, e.g. [46-54]. However, for such a theory to look close to nature one would like to satisfy the relations (2.14),(2.17) or at least satisfy them approximately. This is not automatic and, for example, in the AdS/QCD setup these relations are often not satisfied. (If, in some sense, the contribution of the higher resonances is small, the form factor will be dominated by the rho meson, as is the case in QCD. This often occurs in AdS/QCD due to the oscillatory behavior of the heavier KK wave functions.) See, for example, [55,56] in which concrete cases were analyzed. Even though the specific relations (2.14),(2.17) do not follow automatically from the AdS/QCD approach, when one fits many quantities the agreement with the data is pretty good. See [57] for a recent detailed analysis and also [58] for another interesting aspect of vector meson dominance analyzed in the context of AdS/QCD.

The result (A.3) (which we argued follows from consistency) is satisfied in models of AdS/QCD in an interesting way. It is translated to a certain completeness relation among the wave functions of KK states. Our point of view is that this is really just a consequence of the high energy behavior of the form factor, which is fixed by certain dimensions of operators in the UV CFT.[15] (We have only discussed asymptotically free theories, but one could render the discussion general.)

[15] We thank J. Maldacena for a discussion that led to this claim.

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